

Tennis Ball Launcher: 1D Projectile Motion

1 Overview

You will measure the launch velocity of a tennis ball launcher two different ways (time-of-flight and maximum height of a vertical launch) and compare the results to see if they are consistent with each other.

2 Learning Goals

This week's lab reinforces the statistics learning goals from Lab 3. In addition, after completing this lab, you should be able to:

- Decide whether to use the standard deviation, σ_x , or the standard error of the mean, $\sigma_{\bar{x}}$, when comparing measurements.
- Decide if two measurements likely agree with one another.
- Report a measurement using the proper number of significant figures and value for the uncertainty.

3 Explore: Setup and Initial Measurements

Place your launcher on the ground near the wall and aim it vertically. Use a level to orient your launcher's casing vertically.

First pull back the handle on your launcher to the first click and then load your tennis ball *pushing it in until you feel it reach the back of the chamber (you will feel some resistance)*. Pull the trigger and observe the motion. Never put your face in front of the launcher!

Be careful to make your launches as consistent as possible. Consider having just one person in your group operate the launcher throughout this activity, doing exactly the same things each time.

Practice making two different measurements: (1) the time it takes the ball to go up and come back down to its initial position (front of launcher), and (2) the maximum height of a vertical launch using a meter stick positioned behind the launcher. Note: Make sure to always measure the height of the top of the ball, as that will be important in later measurements.

Record a few notes in your lab notebook:

- Using the stylus, make a sketch of the setup and motion. Indicate the important distances, making note of which part (top, middle, bottom) of the ball you are observing.
- Roughly, what is the time-of-flight?
- Roughly, what is the maximum height?
- What is your procedure to make launches consistent?
- What are the sources of uncertainty?

4 Predict: Velocity Formulas and Air Resistance

You will use two formulas to compute the launch velocity from your measurements. For the maximum height method, conservation of energy gives the initial velocity v_0 in terms of the maximum height h_m :

$$mgh_m = \frac{1}{2}mv_0^2$$

$$2gh_m = v_0^2$$

$$\boxed{\sqrt{2gh_m} = v_0}$$

Here m is the mass of the ball, and g is the acceleration due to gravity. In Boulder, $g = 9.796 \text{ m/s}^2$.

For the time-of-flight method, use the 1D kinematic equation for position with constant acceleration:

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

We can define the launch point as zero, so that both the initial position $x_0 = 0$ and the final position $x = 0$. We can also set the acceleration $a = -g$, where the negative sign means the acceleration is downward. We then have:

$$0 = v_0t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 = v_0t$$

One possible solution is $t = 0$, which represents the moment the ball was launched. But we are interested in the other solution, when the ball comes back down again, which we can find by dividing both sides by t :

$$\boxed{\frac{1}{2}gt = v_0}$$

Do the following in your lab notebook:

- Write down both launch velocity expressions (the boxed equations above) so you have them as a reference.
- Based on your initial observations, which method do you think is more *accurate*, and why?
- Air resistance is complicated, but as a rough estimate the magnitude of the force (in Newtons) is $F \approx 0.001 \cdot v^2$, where v is the speed of the ball (in meters per second). v is the instantaneous speed and is not necessarily constant in time. However, to estimate an upper bound, you can use the largest speed the ball would have during the experiment. Comment on the impact of air resistance to your measurement:
 - Compare air resistance to the weight of the ball. (The mass of a tennis ball is approximately 57 g.)
 - Will ignoring air resistance produce a significant error in the calculated v_0 ?

5 Gather and Analyze Data: Launch Velocity

Embed a new Excel spreadsheet in your notebook (**Insert > Spreadsheet > New Excel Spreadsheet**) and gather new data (at least 10 measurements for each method). You may want to “warm up” the rubber band by firing the launcher a few times before taking your data.

5.1 Reporting with Significant Figures

When reporting a measurement’s uncertainty, round it to one significant figure. For example, if a calculation yields $\sigma_{\bar{x}} = 0.007243 \text{ m}$, report $\sigma_{\bar{x}} = 0.007 \text{ m}$.

There is one common exception: when the uncertainty's leading digit is 1, keep two significant figures. Rounding 0.0143 m down to 0.01 m would discard almost a third of its value, so report 0.014 m instead. The lab app follows this same rule when it displays fit results.

Then avoid extra digits in the reported mean: the last digit in the mean should be in the same decimal place as that of the uncertainty. For example, if a calculation yielded $\bar{x} = 3.40014$ m and the calculated uncertainty was $\sigma_{\bar{x}} = 0.007243$ m, then report the answer as $\bar{x} = 3.400 \pm 0.007$ m. Keeping the 0.00014 in the mean would be nonsensical because the uncertainty shows we are not even sure about the 0 in the thousandths place!

Rules for significant figures

- All non-zero digits (1,2,3,4,5,6,7,8,9) are significant.
- All zeros between non-zero digits are significant. (e.g., 101 has three sig. figs.)
- Zeros that set the decimal point are not significant. (e.g., 0.002 has one sig. fig.)
- Trailing zeros which don't set the decimal point are significant. (e.g., 27.120 has five sig. figs.)

Write a formula in your spreadsheet to compute the launch velocity for each data point you take (see [Lab 1](#) if you need a refresher on writing formulas). Compute the average, standard deviation, and standard deviation of the mean of these launch velocities. Record your final results for v_0 using both methods, with proper significant figures.

6 Compare: Two Methods for Launch Velocity

It is, of course, unlikely that your two independent methods (call them A and B) yielded exactly the same values for $\overline{v_0}$. If you were asked to quote a launch velocity, which method would you quote? Is one method "better" than the other? Is one method "right" and the other method "wrong"? The more scientific way to handle these questions is to make a statement like "the two methods yielded results which have a high (or low) likelihood of being consistent with each other", or "more data is required to determine the consistency of these two methods."

$$| \overline{v_{0,A}} - \overline{v_{0,B}} | < \sqrt{\sigma_{\overline{v_{0,A}}}^2 + \sigma_{\overline{v_{0,B}}}^2} \Rightarrow \text{likely agreement}$$

$$| \overline{v_{0,A}} - \overline{v_{0,B}} | > 3\sqrt{\sigma_{\overline{v_{0,A}}}^2 + \sigma_{\overline{v_{0,B}}}^2} \Rightarrow \text{likely disagreement}$$

otherwise $\Rightarrow$ more data required

where $\overline{v_{0,A}}$ is the mean velocity using measurement method "A," and $\sigma_{\overline{v_{0,A}}}$ is the uncertainty in that mean velocity.

In your notebook:

- Compute $| \overline{v_{0,A}} - \overline{v_{0,B}} |$ and $\sqrt{\sigma_{\overline{v_{0,A}}}^2 + \sigma_{\overline{v_{0,B}}}^2}$ and make a statement about the consistency of your data from your two methods.
- If you observe likely disagreement, what do you think might have gone wrong?

7 Predict: Hand-Hit Probability

Recall from Lab 3 that in a "normally" distributed, large data set, 68% of the measurements will lie in the range from $\bar{x} - \sigma_x$ to $\bar{x} + \sigma_x$. In addition, note that a normal distribution is symmetric about its mean. If you

were to place your hand at a height $h = \overline{h_{\max}} + \sigma_{h_{\max}}$, what percentage of the time should you expect the ball to strike your hand?

Record your prediction along with your reasoning in your notebook.

8 Discuss: Comparing Predictions with Your Pod

Pair up with another group in your pod, and discuss your reasoning for the prediction in Section 7.

If you decide to make any changes to your prediction or reasoning, record them in your lab notebook.

If you made a mistake, do not remove what you have already entered in your notebook. Just make a note indicating your correction. You won't be marked off for recognizing your mistakes; this is part of learning in the lab!

9 Gather and Analyze Data: Testing the Statistical Prediction

Now you will test out your prediction! (Remember to "warm up" your launcher.)

Put your hand at h above the launcher, perform 10 launches, and count the number of times the ball hits your hand.

Record the results in your notebook, and whether they matched your prediction.

Input your number of hand hits into the spreadsheet on the shared tablet on the lab wall connected to the TV.

Record the results of your lab section as a whole in your notebook. If you average the results of your entire section, does that value match your prediction?

10 Draw Conclusions: When to Use σ_x vs. $\sigma_{\bar{x}}$

In your notebook, do the following:

- Record a statement about when you should use σ_x .
- Record a statement about when you should use $\sigma_{\bar{x}}$.
- Which is more likely to match a prediction: a single group's results or the results of a lab section as a whole? Explain your reasoning.

11 Reflect

Briefly, record your responses to the following in your lab notebook:

- In the next lab, you will begin by re-measuring the launch velocity of a launcher (perhaps not the same one!) using the maximum height method. What will you do differently next week when you repeat the maximum height measurement?
- Describe a memorable moment during today's lab.
- Why was it memorable?