Parallel Plate Capacitor with a Dielectric Material

1 Overview

You will determine the dielectric constant of a paper by measuring the capacitance of a parallel plate capacitor with sheets of paper between the plates.

> Start a new notebook
>
> Before you begin, start a new notebook for this lab and title it `Lab 7`: in the PHYS 1140 Notebook, tap the **Notebook** menu, then **New notebook…**. See Start a Notebook for Today's Lab.

2 Learning Goals

After completing this lab, you should be able to:

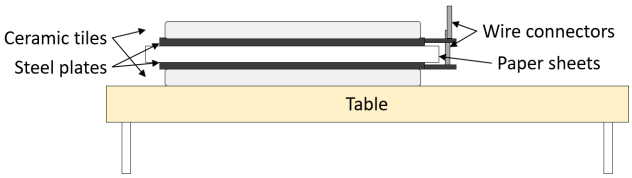
- Identify and address a systematic effect.
- Modify the physical model to take a systematic effect into account.

> Note
>
> For this lab, we're not including error analysis in order to focus on other learning goals. You are welcome to practice your error analysis skills in this activity, but doing so is not required.

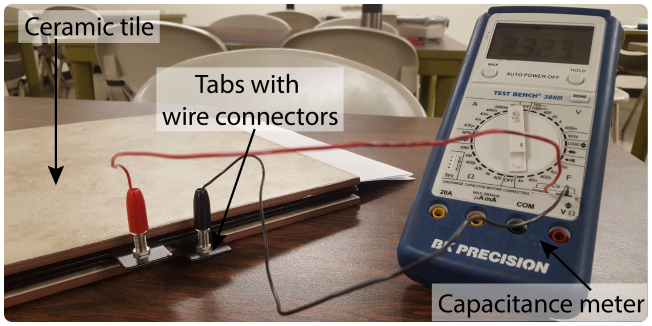
3 Explore: Building a Capacitor

Build a capacitor consisting of two metal plates separated by some sheets of paper (a dielectric material), as shown in Figure 1. The ceramic tile and weights are necessary to make sure the paper is flat while you make the measurements. Connect the plates to a capacitance meter with wires. To set the meter to measure capacitance, turn the large central dial to the region marked `F` (for Farads), and set it to `4n` (the lowest range setting). Throughout this activity if the capacitance goes above 4 nF during your measurements, turn the dial to the next highest setting (`40n`).

> Sketch a diagram of your setup in your lab notebook.



(a) Plate setup



(b) Paper sheets

Figure 1: Parallel plate capacitor setup. The paper sheets in the diagram are not drawn to scale.

Spend a few minutes exploring the following questions.

Make some notes in your lab notebook:

- What happens if you touch the plates during a measurement, or wave your hand near it?
- How long does it take for the meter reading to stabilize?
- What happens (qualitatively) if you use more sheets of paper?

4 Gather and Analyze Data: Initial Measurement of κ

You will measure the dimensions of the plates, the thickness of the paper, and the capacitance. For the paper thickness you will use calipers, a tool for measuring the distance between opposite sides of an object more precisely than a ruler can. The calipers for this activity are dial calipers: the dial reads in millimeters with minor ticks every 0.1 mm, and a dot at the top of the linear scale marks each full 10 mm the dial has turned. The linear scale also has inch markings, but work in metric to take advantage of the dial's precision. Calipers can measure both outer ("Outer Diameter," OD) and inner ("Inner Diameter," ID) distances; Figure 2 shows each.

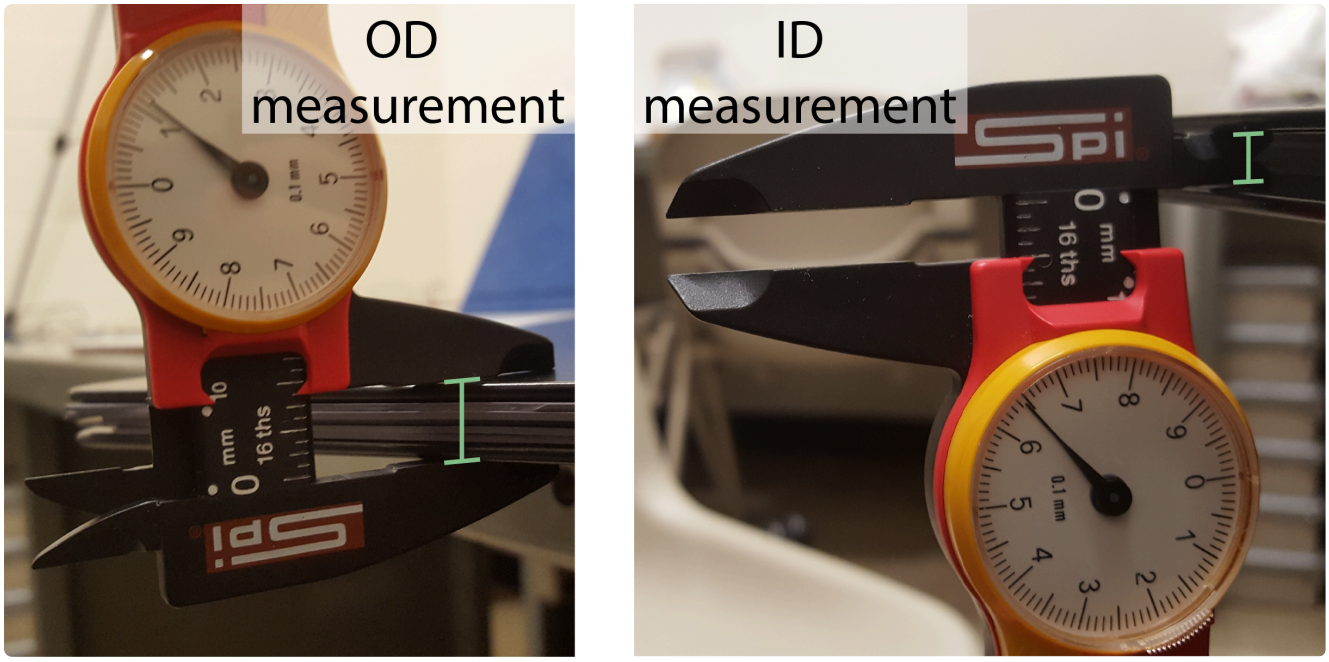


Figure 2: Photos of two ways to make a measurement using calipers: an Outer Diameter (OD) measurement and an Inner Diameter (ID) measurement.

Record the following measurements in your notebook (with units):

- Using a tape measure, measure the length l and width w of the metal plates.
- Using calipers, measure the thickness d_p of a sheet of paper. One piece on its own is too thin for the calipers, so measure many sheets (10 or more) together and then divide by the number of sheets.
- Set up your parallel plate capacitor with one sheet of dielectric and measure the capacitance C.

In the prelab, you saw the formula for the capacitance of an ideal parallel plate capacitor (Figure 3),

$$C = \epsilon \frac{A}{d} = \kappa \epsilon_0 \frac{A}{d},$$

where $A = \text{length} \times \text{width}$ is the area of one plate, d is the distance between the plates, ϵ is the permittivity constant for a material, ϵ_0 is the permittivity of free space, and κ is the dielectric constant or relative permittivity for a material. $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

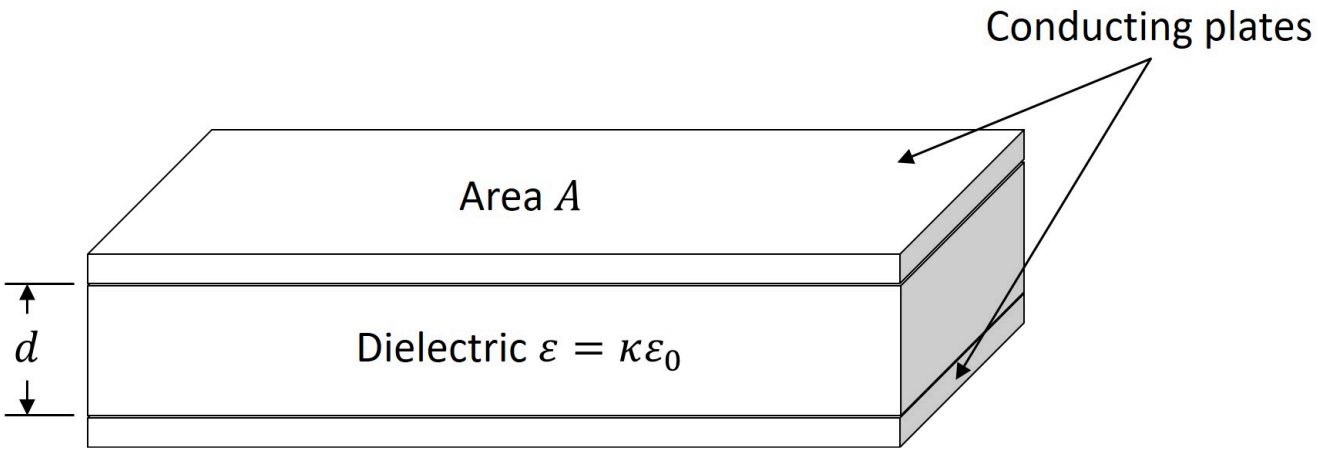


Figure 3: Parallel plate capacitor

Solve this expression for κ, the dielectric constant, in terms of l, w, d_p, and C. You should find

$$\boxed{\kappa = \frac{Cd}{\epsilon_0 A}}$$

Use this to compute κ of the paper based on your measurements. (You should have also done this in the prelab.)

> Record your formula and your computed value for κ in your lab notebook.

5 Predict: Does κ Change with Thickness?

To do better than a single measurement, you can measure κ using different numbers of paper sheets stacked together. Do you expect κ to change based on the number of sheets used?[1]

> Write your answer to that question in your lab notebook, and explain your reasoning.

6 Gather and Analyze Data: κ vs. Plate Separation

> Measure C for $N = 2, 3, 4, 5$ and then (important!) jump up to one final measurement with 10 sheets of paper. Multiply the thickness of a single sheet, d_p, by N to calculate the plate separation d for each measurement. Add a spreadsheet to your notebook (tap **Spreadsheet** in the tool bar) and, for each measurement individually, compute κ. Make sure your calculations are included. Make a plot of κ as a function of d with labeled axes, units, and a caption.

To make this plot, choose any cell of your spreadsheet so that the bar above it shows the cell's name, tap **Add plot** in that bar, and type the range of your d column in **X cells** and the range of your κ column in **Y cells** (a range is written like `B1:B6`). Check those boxes before you tap **Add plot**: the notebook fills them in from the cells you had selected, and three selected columns are always read as X, Y, and error bars. See Plotting in Your Notebook in Lab 2 for the rest.

7 Discuss: Identifying the Systematic Trend

> Answer the following questions in your notebook:
>
> - Is there a trend in your data? That is, does κ tend to increase or decrease as N changes?
> - Does your plot of κ as a function of N support your prediction in Section 5? Does this appear to be a situation where random errors or systematic issues dominate? Why?
> - What might be some possible causes of the discrepancy? (Don't spend too much time on this last question, just think/write a bit. We will spend the rest of the lab investigating this!)

[1] κ is an *intrinsic* property of a material, like density or color, that doesn't change based on how much material is present. On the other hand, *extrinsic* properties, like mass or volume, change based on how much material is present.

8 Compare: Sharing Results with Your Pod

Discuss your answers to the questions in Section 7 with other members of your Pod. If you need to revise your original answers, do so now. Leave your original thoughts.

9 Analyze Data: Revised Model with Stray Capacitance

The phenomenon of κ changing with N is an example of a "systematic effect." One probable cause is that there are additional sources of capacitance beyond the simple plate-paper model. We can address this systematic effect by representing that extra capacitance in our model as an additional capacitor in *series* with the plate-paper capacitor.

Extending the model to include another capacitance, C_0, in series with the ideal parallel plate capacitance C_{ideal}, the measured capacitance C_m is found by summing the inverses,

$$\frac{1}{C_m} = \frac{1}{C_{ideal}} + \frac{1}{C_0}.$$

Replacing C_{ideal} with the expression $C = \kappa\epsilon_0 A/d$ from the previous section and rearranging gives our revised result,

$$\boxed{\kappa_{revised} = \frac{d}{\epsilon_0 A}\left(\frac{1}{C_m} - \frac{1}{C_0}\right)^{-1}}$$

where C_m is the measured (experimental) value and C_0 is a parameter that must still be determined. (The algebra is just a few steps — feel free to work through it yourself if you're curious where the result comes from.)

9.1 Finding C_0 by Guess and Check

Re-analyze your data using the revised model. Since we cannot easily measure C_0 directly, we use the fact that we expect κ to remain constant as N changes, and adjust C_0 to make κ as constant as possible. (Dielectric constants of typical materials are in the range of 1–10.) A good *initial guess* for C_0 is the capacitance without any paper in place.

Keep working in the same spreadsheet you added above — it already holds your measurements. Add a single cell to hold the value of C_0 and enter your initial guess, then add a new column that recomputes κ for each measurement with the revised formula, referring to the C_0 cell. Make a new plot of the recomputed κ vs. d the same way, with the new κ column in **Y cells**; it redraws each time you change a cell it uses. Change the value in the C_0 cell until this plot is as flat as possible.

You can copy the revised κ formula into your spreadsheet instead of typing it by hand. Choose an empty cell so that the bar above the spreadsheet shows its name, then tap it again, or press **Enter**: the cursor starts blinking in the cell. Then paste the formula (**Ctrl+V**).

```
=(d/(E0*A))*(1/Cm - 1/C0)^-1
```

Replace each variable with the cell reference for your experiment:

- **d** — the cell with your plate separation d (in meters)
- **E0** — the vacuum permittivity, 8.854×10^{-12} F/m
- **A** — the cell with your plate area (use `$` signs, e.g. `$B$2`, so this single cell stays fixed)
- **Cm** — the cell with your measured capacitance (in farads)

- **C0** — the cell with your stray-capacitance guess (use `$` signs so this single cell stays fixed)

Then press **Enter**. The cell shows `#NAME?` until every variable is replaced with a cell: tap the cell, and the notebook says which name it does not know. If the cell shows the formula itself as text, the cursor was not in the cell when you pasted: clear the cell and paste again.

Record your results using the revised model in your notebook, including a new plot of κ vs d (with labeled axes, units, and a caption) and your final value of C_0.

Note

There are more sophisticated methods than guessing and checking for determining unknown parameters like C_0 from data. We will use one such method, curve fitting, later in this class.

10 Discuss: Interpreting C_0

Answer the following questions in your notebook:

- In your own words, what does C_0 represent? What are some possible physical phenomena that might be contributing to it?
- Do systematic effects remain in your measurement of κ? If so, what are some other possible causes for those effects?

11 Draw Conclusions: κ and Systematic Effects

Record a few conclusions in your notebook:

- Based on your results from the revised model, what would you report for the dielectric constant κ of paper?
- What is the difference between systematic effects and statistical (or random) uncertainty?

12 Reflect

Briefly, record responses to the following in your lab notebook:

- Describe a memorable moment during today's lab.
- Why was it memorable?

When you are done, export your notebook and upload both files to Canvas: see Turn In Your Notebook. Then log out of the tablet computer and check it back in with your TA, along with its pen.